

Bose-Einstein condensation and a free-market economy

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Abstract - Financial systems can be described using a gas analogy. It is possible to calculate equilibrium statistical distributions following from the model. The structure of these distributions depends crucially on average income, democracy, the standard of life and other features associated with political and economic structures of the market economy at equilibrium and may be described by a few parameters, only. One of these parameters can be called a temperature - connected with an average money which economic agents have. The Bose-Einstein distribution of money arises when appear more economic agents who use the largest part of their money in their individual single trading processes.

I. INTRODUCTION

In the past couple of decades, a large number of physicists have started exploring problems which fall in the domain of economic science. Papers on econophysics have been published primarily in journals devoted to physics and statistical mechanics, rather than in leading economics journals. Mainstream economists have generally been unimpressed by this work. Physics is seen by them to be primarily a study of interaction between simple elements, while economics deals exclusively with rational agents, able to formulate complex strategies to maximize their individual welfare. However, the current worldwide crises revealed the inadequacies of mainstream economic theory.

This has now led some economists to believe that new approach of looking at the economic problems cannot be simply ignored [1]. Bringing together the techniques of statistical physics and nonlinear dynamics to study complex systems along with the ability to analyze large volumes of data with sophisticated statistical techniques, econophysics provides a promising alternative to study economic phenomena.

The paper is organized as follows: Section II describes some risks in financial transactions and the random nature of a market, as the introduction for further sections. Section III deals with the linkage between gas dynamics and economy and briefly describes some interesting analogy. Section IV briefly describes Bose – Einstein Condensation (BAC), while section V deals with the application of the BAC concepts to the financial crisis. Finally, section VI is the conclusion.

II. RISKS AND FINANCIAL ASSETS

There are riskless and risky financial assets. An example of a riskless financial assets is a bank account since there is a guarantee to receive usually fixed interest rate r , regardless of the market situation. Banks operate in a way that they borrow from those who have the money but are not willing to take risks, and lend at higher interest rates to those who need money. Banks can reduce their overall risk by diversifying their lending [8].

On the other hand a company can also raise capital by issuing stocks or shares. Stocks are risky financial assets because their prices are subjected to unpredictable fluctuations. This is what makes stocks attractive to investors who seek to buy low and sell high.

The stocks exchange is usually done in the organized way, e.g. by the New York Stock Exchange – NYSE or the São Paulo Stock Exchange – BOVESPA. The stock exchanges have indexes that are linked with average behavior of some market. The stock represents a small piece of a company and reflect the net worth of this company.

The basic problem in risky financial market is to predict the future on the basis of present information. If a new information is revealed that might affect the company's future performance and the stock price will vary accordingly. But the future price of a stock is always uncertain in some degree. The behavior of a market is erratic. Although stock prices are rather unpredictable this does not mean that they cannot be modeled in a probabilistic fashion.

Let S be a price of a given stock so we can write an equation for the rate increase of S :

$$\frac{dS}{dt} = R(t)S$$

where $R(t)$ is the rate of return of the stock. According to [8], $R(t)$ could be separated into two components:

- a predictable mean rate return, denoted by μ ,
- a fluctuating or noisy term $\xi(t)$ responsible for uncertainty in the stock price.

Now, we have:

$$R(t) = \mu + \xi(t),$$

and finally:

$$\frac{dS}{dt} = [\mu + \xi(t)]S$$

One of the best models for “noise” is the white noise, so that it is understandable that Brownian motion and white noise play very important role in finances and economy.

It is obvious that there is an analogy between finances and economy on one side and gas model, BAC theory on the other side. This analogy may be of a great importance for small and medium sized enterprises.

III. GAS ANALOGY

In econophysics, theories like statistical mechanics are applied to large populations of people interacting with each other by exchanging money, goods, and services. In the gas model, people exchange money in random interactions, much as atoms exchange energy when they collide.

While economists' models traditionally regard humans as rational beings who always make intelligent decisions, econophysicists argue that in large systems the behaviour of each individual is influenced by so many factors that the net result is random, so it makes sense to treat people like atoms in a gas [2]. Yakovenko and his colleagues proposed that the Maxwell-Boltzmann-Gibbs formula can be applied by treating money as analogous to energy [3].

IV. BOSE-EINSTEIN DISTRIBUTIONS

It is generally accepted that the statistical distributions of variations of prices of financial assets (shares, indices, commodities, exchange rates) are power laws [4], and (exponentially) truncated power laws [5]. In the article "Bose-Einstein Condensation in Financial Systems" [7], was shown that financial systems can be described as condensates, similar to Bose-Einstein Condensates (BEC). A Bose-Einstein condensate is a state of matter of a dilute gas of weakly interacting bosons confined in an external potential and cooled to temperatures very near absolute zero (0 K or -273.15°C). Under such conditions, a large fraction of the bosons occupy the lowest quantum state of the external potential, at which point quantum effects become apparent on a macroscopic scale. The Bose-Einstein distribution for the occupation of energy states is also an exponentially truncated power law. The power law exponents in finance markets are however different from those in BECs. Despite of the significant differences in power law exponents, the fact that the distributions both in financial markets and BECs

follow an exponentially truncated power law is remarkable.

Another indication that the financial markets are related with BECs is that both systems are partially random, and partially coherent. The atom collisions in classical gases are completely random (constrains being just the energy and momentum conservation), which leads to Maxwell-Boltzman distributions. The particle collisions in bosonic gases are selective, in that the atoms after collisions prefer to choose occupied states, due to the bosonic enhancement effect. In [6] Staliunas argues that the events in finance markets are somehow motivated and that that motivation in general brings order and coherence into a system: "The bosonic enhancement is responsible for the coherence in atomic (or photonic) condensates, in that the (quantum) particles tend to choose occupied states.

In finance, one obvious behavior scenario is that most market participants tend to invest like the others participants, i.e., to occupy more attractive, more popular, in general already occupied, states. This is due to a choice of investing strategies according to the opinion of majorities. This is also due to a condensation of investors into investment groups, with common investment strategies."

Staliunas proposed the following model: each market participant occupies states in a two dimensional space:

$$X=(m,s),$$

where m is the amount of money, and s is the amount of shares in possession. Assuming that two particles (two market participants) involved in a collision (a deal) were initially in states:

$$X_1=(m_1,s_1),$$

$$X_2=(m_2,s_2)$$

with the average occupations n_1 and n_2 , and that after collision they occupy new states:

$$X'_1=(m'_1,s'_1),$$

$$X'_2=(m'_2,s'_2)$$

with average occupations n'_1 and n'_2 , the probability of the above collision is:

$$n_1 n_2 (1+n'_1)(1+n'_2).$$

Taking into account conserved quantities (money) and the fact that probability of the transition in the reverse direction is equal to that of the forward transition, it was obtained the following solution:

$$\frac{n(m, s)}{1 + n(m, s)} = \exp[\beta(\mu - m - s)].$$

Here, μ is having the meaning of a chemical potential indicating the level of condensation in the system, and $\beta=1/(kT)$ is having the meaning of inverse temperature. This equation easily leads to Bose-Einstein distribution:

$$n(m, s) = (\exp[\beta(-\mu + m + s)] - 1)^{-1}.$$

In this analogy temperature kT in finance markets would have a meaning of average richness of market participants. Concluding, based on the assumption that the individual market participants tend to cluster, and to behave according to the opinion of majority, one can derive a Bose-Einstein distribution. However, taking into account above assumption together with the fact that the market participants seek for profit, Staliunas obtained exponentially truncated Pareto and Levy distributions with power exponents corresponding well to the ones observed in financial markets.

V. BEC AND FINANCIAL CRISIS

Interesting results are pointed by Kurten and Kusmartsev in [7]. They have studied a simple model of the market with a fixed number of N economic agents interacting in pairs. Originally before a start of the trading processes all agents have different amounts of money with equal probability.

At each time step a randomly chosen agent i acts as a buyer, while the connected randomly chosen agent j acts as the seller. As the result the wealth of the i -th agent decreases as:

$$m_i(t + 1) = m_i(t) - pm_i(t),$$

while for the j -th agent the money increases as:

$$m_j(t + 1) = m_j(t) + pm_i(t).$$

The parameter p is associated with the fraction of money used in individual financial transactions. It was shown that the safe market economy arising at small values of $p < 1/2$ corresponds to a gamma distribution where the majority of economic agents have approximately the same amount of money. In such a case

of the safe market economy agents are very cautious to use all their money in a single trading event.

While, when $p > 1/2$ the agents are gambling, that is they use the largest part of their money in their individual single trading processes. This unavoidably leads to a fact that some significant fraction of economic agents will loose their money and condense into a state without money. When this happens there arises BEC state, where the majority of agents have zero, or have essentially no money. The next participation of these agents in the market is prohibited unless they will loan the money and receive the debt or be very lucky to sell successfully their property or wealth. On the other hand people get poorer and poorer and most of the people have no money to trade. On average there are few rich people on the market, i.e having a lot of money. But most of the people remain poor for the long time period.

Such a distribution may effectively describe a formation of financial crisis as a Bose-Einstein Condensation phenomenon as well as economic growth of human society.

VI. CONCLUSION

In summary the financial crisis of 2007-2010 started with the US subprime mortgage crisis and still experts are placing different weights upon particular causes of the crisis. The complexity and interdependence of many of the causes, as well as competing political, economic and organisational interests, have resulted in a variety of point of views describing the origin of the crisis.

Econophysics provides a promising alternative to study financial crises. It shows that the model of a stable market can be achieved when the economic agents use a small fraction of their money in the financial operations. On the other hand it was demonstrated that market with gambling trading is very risky and unavoidably leads to a fact that some significant fraction of economic agents will loose their money and condense into a state without money.

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